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$$C=\sqrt{\frac{1}{2}}\cdot\left[1-\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{7}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{1}{11}+\dots\right]$$

which gives $C=0.59922$.

Now if x be taken equal to 2, yet for this small number the first two terms of the series will afford tolerable accuracy. Let $x=2$, which corresponds to the abscissa $\cos ix=3.7711$, or to the abscissa counted from the vertex $=2.7711$; then $s=\sqrt{\tfrac{1}{2}}.(7.38905-\tfrac{1}{6}.\tfrac{1}{403.42})-0.59922=4.6175$.

As x increases the value of s approachcs more and more to

$$s=e^{x}.\sqrt{\tfrac{1}{2}}-0.59922.$$

In a similar manner the rectification of the hyperbola whose equation is $a^2x^2-b^2y^2=a^2b^2$ may be performed. Also the rectification of the ellipse. Putting $z=\sin(a+ix)$ we have the equation of the hyperbola, as before. The element of the curve here becomes

$$ds=\sqrt{(\cos^2 ix-\tan^2 a\sin^2 ix)}dx$$

$$=\sqrt{[\tfrac{1}{4}(e^{2x}+2+e^{-2x})+\tfrac{1}{4}\tan^2 a(e^{2x}-2+e^{-2x})}dx$$

$$=\tfrac{1}{4}e^{x}\sqrt{\left[1+\frac{c}{e^{2x}}+\frac{1}{e^{4x}}\right]}dx,$$ denoting by c the quantity $2\left[\dfrac{1-\tan^2 a}{1+\tan^2 a}\right]$

Hence $$ds=\tfrac{1}{4}e^{x}\left[1+\frac{1}{2}\left(\frac{c}{e^{2x}}+\frac{1}{e^{4x}}\right)-\frac{1}{2}\cdot\frac{1}{4}\left(\frac{c}{e^{2x}}+\frac{1}{e^{4x}}\right)^2+\,..\right]dx$$

and integrating

$$(15)\quad s=\frac{1}{4}\left[e^{x}-\frac{c}{2}\cdot\frac{1}{e^{x}}-\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{e^{3x}}+\frac{1}{2}\cdot\frac{1}{4}\left(\frac{c^2}{3e^{3x}}+\frac{2c}{5e^{5x}}+\frac{1}{7e^{7x}}\right)+\,..\right]$$

which series also converges very rapidly.

In conclusion I will state that all integrals of the form $\int\cos^m x\sin^n x\,dx$ and most of the binomial integrals $\int x^m\left(a+bx^n\right)^{\frac{p}{q}}dx$ can be found in a more simple and elegant way than taught in the text-books, by employing imaginary circular functions.

Paradox for Students in Analytical Geometry.—The equation of a given curve of the third degree being

$$Ax^3+Bx^2y\dots+Ex^2+Fxy\dots+Hx\dots+K=0,$$

if $P_1,(x_{,}y_{,})$, is a point on the curve, the equation

$$Ax_{,}^2x+\tfrac{1}{3}B(2x_{,}y_{,}x+x_{,}^2y)\dots+\tfrac{1}{3}E(x_{,}^2+2x_{,}x)+\tfrac{1}{3}F(x_{,}y_{,}+x_{,}y+xy_{,})$$
$$\dots+\tfrac{1}{3}H(x_{,}+2x)\dots+K=0\dots(1)$$

represents the tangent to the curve at the point P_1.

If P_1 is not restricted to be a point of the curve the straight line (1) is called the *polar*, with respect to the given cubic, of the point P_1, as in the case of the similarly formed equation derived from the equation of a curve of the second degree. The equation

$$Ax^2x_{,} + \tfrac{1}{3}B(2xyx_{,} + x^2y_{,}) \ldots + \tfrac{1}{3}E(x^2 + 2xx_{,}) + \tfrac{1}{3}F(xy + xy_{,} + x_{,}y)$$
$$\ldots + \tfrac{1}{3}H(x + 2x_{,}) \ldots + K = 0 \ldots (2)$$

formed by interchanging $x_{,}$ and x, $y_{,}$ and y, represents a conic of which P_1 is said to be the pole. we may call it the *conic* of P_1 with respect to the given cubic.

From formulæ (1) and (2) we deduce properties of polars and conics with respect to a cubic analogous to those of reciprocal polars with respect to a curve of the 2nd degree.

1. If a point P_2 is on the polar of P_1, then P_1 is on the conic of P_2, and conversely; that is to say, the conics of all the points on the polar of a fixed point pass through that point, also the polars of all the points on the conic of a fixed point pass through that point.
2. Therefore to find the pole of a conic we may draw the polars of two of its points, their intersection determining the pole required.
3. Also, to find the pole of a straight line we may draw the conics of two of its points, their intersection determines the pole required.
4. But these conics intersect in two or more points therefore a straight line has two or more poles.
5. The several points in which two conics intersect have the same polar, namely the straight line joining the poles of the two conics.
6. But two conics may be drawn through any two points; therefore any two points, that is all points have the same polar.

Now equation (1) cannot represent the same straight line whatever the values of $x_{,}$ and $y_{,}$; therefore the conclusion in (6) is false. Point out the fallacy in the above reasoning. — WM. WOOLSEY JOHNSON.

PROBLEM — Given four points (no one point lying within the triangle formed by the other three), to construct geometrically the axis and focus of the parabola passing through them.

Being already acquainted with methods for constructing the axes of the ellipse (or hyperbola) passing through any five given points, and of course with that for the circle passing through any three points, the problem just given has baffled all attempts on my part for some time.

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